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An experiment on axisymmetric flow of liquid sodium in a magnetic field

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With 4 figures in the text

SUMMARY

An axisymmetric motion has been generated by a rotating disc in a cylindrical vessel containing 58 litres of liquid sodium and placed in an external magnetic field. Measurements of the internally induced magnetic fields indicate the existence of a toroidal component which circulates around the axis of symmetry and a poloidal component which is restricted to planes through the same axis. The results agree with the elementary theory, as far as orders of magnitude are concerned.

There has been observed neither a formation of instabilities in the twisted field, nor an operation as a selfexciting dynamo; the velocities in the experiment are too small and the degree of asymmetry too low to produce such phenomena.

I. Introduction

The possibility for a self-exciting dynamo to exist inside a singly connected fluid mass has been discussed by a number of theoretical investigators. Reviews on the subject are given by Bullard and Gellman (1954), Elsasser (1955) and Cowling (1957) and recently some new ideas have been put forward by Herzenberg (1957). The problem mainly consists of finding certain types of fluid motions and magnetic fields which sustain each other by means of their mutual interaction. Cowling (1934, 1957) has pointed out that a magnetic field cannot be maintained by fluid motion in an axisymmetric case. In fact, the field must have a complicated form which is difficult to picture. It has been suggested by Alfvén (1950) that the magnetic energy can be increased if internal motions twist the magnetic field lines and cause instabilities, and that this may form an integrating part of the dynamo mechanism. The external field of the dynamo, however, will be severly limited when the fluid mass is singly connected and has infinite electrical conductivity (Bondi and Gold 1950).

In this paper is described an experiment on a type of axisymmetric flow of liquid sodium in a magnetic field. The experiment does not aim directly at the dynamo mechanism. Its main purpose is merely to investigate the order of magnitude of the internal magnetic fields which are induced by motions inside a fluid conductor of laboratory dimensions. It also gives some aspects on the possibility to study the instability of a twisted magnetic field.

II. The apparatus

As shown in Fig. 1, a vessel containing 58 litres of liquid sodium has been used in the experiment. The fluid is set into motion by a rotating copper disc with radially attached strips, the disc being driven by a motor in clockwise or anti-clockwise direction. Similar strips are fixed to a copper plate at the bottom of the vessel. An axisymmetric magnetic field **B** can be generated by a coil below the vessel. A probe coil in the shape of a vertical frame is attached to a holder which can be turned around its axis and moved in vertical direction. This makes possible the measurement of a magnetic field directed perpendicularly to the probe in a region between the disc, the bottom, the inner surface of the vessel and a line 4.3 cm distant from the axis of symmetry. The measurements are made by switching on the supply current through the coil below the vessel and reading the corresponding deflection on a flux meter connected with the probe coil.

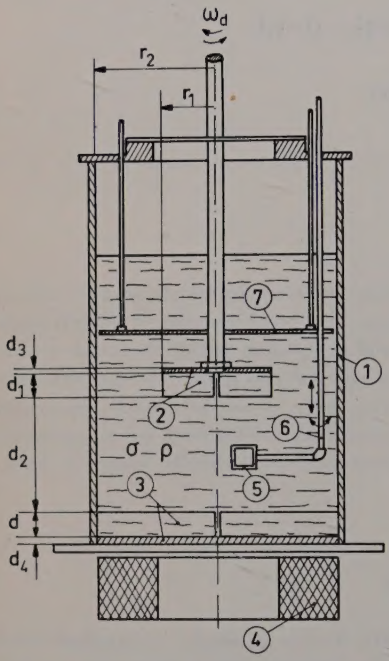

Fig. 1. 1. Cylindrical vessel containing 58 litres of liquid sodium. 2. Copper disc with radially attached strips. 3. Bottom plate of copper with similar strips. 4. Coil which generates the external magnetic field. 5. Probe consisting of a coil in the shape of a frame. 6. Holder of the probe. 7. Roof which reduces the motion of the upper layers. The dimensions are: $r_1 = 9$ cm; $r_2 = 20$ cm; $d_1 = 4$ cm; $d_2 = 19$ cm; $d_3 = 2.5$ cm; $d_4 = 1.0$ cm. At 120°C the conductivity of sodium is $\sigma = 1.03 \times 10^7$ ohms^{-1} m^{-1} and of copper $\sigma_d = 4.06 \times 10^7$ ohms^{-1} m^{-1} and the density of sodium $\varrho = 922$ kg/m^3.

III. Experimental results

In the first experiment the copper disc and the bottom plate were used without radial strips. The power input was measured when the disc rotated with angular velocities ω_d between 0 and 42 sec^{-1}. The mean external field B_d in the region of the disc was varied between 0 and 0.02 Vs/m^2 (200 gauss). With $\omega_d = 42$ sec^{-1} the power increased from 84 to 200 watts when a magnetic field $B_d = 0.02$ Vs/m^2 was introduced.

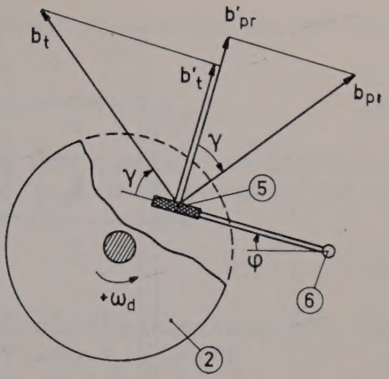

Fig. 2. Definitions of the toroidal field strength b_t, the radial poloidal component b_{pr} and the angle γ. The view is from the top of the vessel (1) in Fig. 1. The probe (5) is turned through an angle φ around the holder (6).

In the second experiment radial strips were used and the power input became about 2700 watts at $\omega_d = 47$ sec^{-1} when the magnetic field was absent. With a field $B_d = 0.02$ Vs/m^2 and at the same angular velocity the power input decreased by about 20 %.

Probe mesasurements were made at a mean magnetic field strength $B_d = 0.0086$ Vs/m^2 in the region of the disc. On the axis of symmetry the field **B** had the strengths 0.0090 Vs/m^2 and 0.028 Vs/m^2 at the disc and at the bottom, respectively. With the probe in a given position the flux meter indicated the deflections $\alpha(+\omega_d)$, $\alpha(-\omega_d)$ and $\alpha(0)$ at the angular velocities $+38.8$, -38.8 and 0 sec^{-1}. The relative strengths of the measured fields b^+ and b^- at anti-clockwise and clockwise rotation become

$$b^+/B = [\alpha(+\omega_d) - \alpha(0)]/\alpha_0 \quad \text{and} \quad b^-/B = [\alpha(-\omega_d) - \alpha(0)]/\alpha_0, \tag{1}$$

where α_0 is the deflection which corresponds to the total strength B of the external field at the point in question. It is supposed that the induced field consists of a toroidal part $\mathbf{b}_t$ which circulates around the axis of symmetry and a poloidal part $\mathbf{b}_p$ which is restricted to planes through the same axis and has a radial component b_{pr}. From the measured values b^+ and b^- we obtain

$$b_t = \tfrac{1}{2}(b^+ - b^-)/\sin\gamma = b_t'/\sin\gamma \tag{2}$$

and

$$b_{pr} = \tfrac{1}{2}(b^+ + b^-)/\cos\gamma = b_{pr}'/\cos\gamma, \tag{3}$$

where γ is defined by Fig. 2. The angle φ in the same figure has been varied in the range ±80°.

A rough picture of the experimental results is given in Fig. 3, the right hand half of which shows the undisturbed external field **B** (dashed lines) and the field $\mathbf{B} + \mathbf{b}_{pr}$ modified by the radial poloidal component (full lines). An axial poloidal component b_{pz} cannot be indicated by the probe. The relative magnitude b_{pr}/B of the radial poloidal field is of the order 0.1 in the middle of the right hand half of Fig. 3. The toroidal field $\mathbf{b}_t$ is shown by the circles in the left hand half of the same figure where the radius of a circle is proportional to the relative magnitude b_t/B. A value $b_t/B = 0.25$ corresponds to the largest circle at a distance of 10 cm from the axis and 2.4 cm from the bottom strips.

Neither during the observations of the power input, nor during the probe measurements there was noticed any sign of dynamo action or of instability.

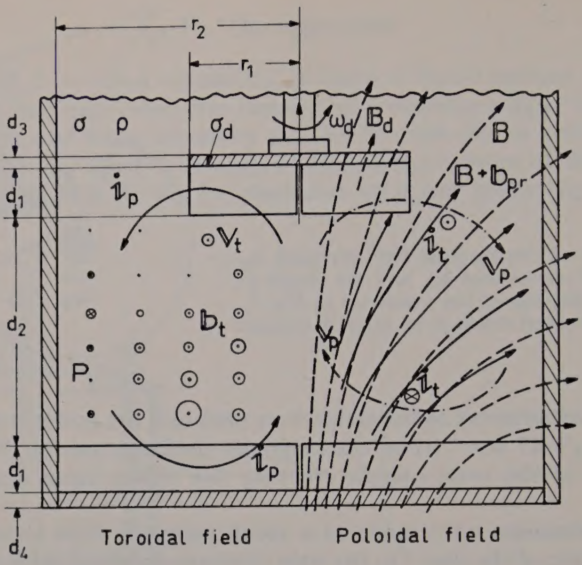

Fig. 3. Right hand half of the figure: The undisturbed external field **B** (dashed lines) and the field $\mathbf{B} + \mathbf{b}_{pr}$ resulting from the external one and the radial poloidal component $\mathbf{b}_{pr}$ (full lines). A poloidal velocity field $\mathbf{v}_p$ (dot and dash lines) is caused by the centrifugal force, as well as a toroidal current system $\mathbf{i}_t$.

Left hand half of the figure: The rotation of the disc generates a toroidal motion $\mathbf{v}_t$ and a poloidal induced current $\mathbf{i}_p$ (full lines). The motion $\mathbf{v}_t$ as well as the induced toroidal field $\mathbf{b}_t$ are modified by the poloidal flow $\mathbf{v}_p$; the magnitude of the arrows describing $\mathbf{b}_t$ are proportional to b_t/B.

IV. Estimations of the induced magnetic fields

The induced toroidal field can be estimated by the following elementary considerations. According to Fig. 3 the rotating disc generates an electromotive force

$$V_p = \tfrac{1}{2}\omega_d r_1^2 B_d. \tag{4}$$

This gives rise to a poloidal current $\mathbf{i}_p$ which passes the disc and is closed in the liquid. The corresponding electric resistance is about

$$R_p = \left[\frac{\log(2r_2/r_1)}{1+\sigma_d d_3/\sigma d_1} + 2d_1 d_2 r_2^2/r_1^2(r_2^2 - r_1^2)\right] \Big/ 2\pi\sigma d_1 \tag{5}$$

when radial strips are used, and about

$$R_p = [\log(2r_2/r_1) + 2\sigma_d d_2 d_3 r_2^2/\sigma r_1^2(r_2^2 - r_1^2)]/2\pi\sigma_d d_3 \tag{6}$$

when the strips are absent; the symbols in eq. (5) and (6) are given by Fig. 3. The poloidal currents are sustained by the power

$$W_p = V_p^2/R_p = R_p I_p^2, \tag{7}$$

where I_p is the total poloidal current. The magnitude of the induced toroidal field is deduced from eq. (4) and (7):

$$b_t/B \approx b_t/B_d \approx \mu I_p/\pi r_2 B_d \approx \mu\omega_d r_1^2/2\pi r_2 R_p. \tag{8}$$

Further, the centrifugal force generates a poloidal flow $\mathbf{v}_p$ and electromotive forces which give rise to toroidal currents $\mathbf{i}_t$, as shown by Fig. 3. At the level of the disc, as well as near the bottom, the electromotive force has the magnitude

$$V_t \approx \pi r_2 \bar{v}_p \bar{B}, \tag{9}$$

where $\bar{v}_p$ and $\bar{B}$ are mean values of the poloidal motion and the external magnetic field in the fluid region. The corresponding electric resistance is about

$$R_t = 2\pi/\sigma r_2 \tag{10}$$

and the power input

$$W_t = 2V_t^2/R = 2R_t I_t^2. \tag{11}$$

The power input is also given by the work performed by the centrifugal force on the poloidal flow:

$$W_t = 2\pi\varrho \iint v_{pr}\,\omega^2 r^2\,dr\,dz. \tag{12}$$

where ϱ is the density, v_{pr} the radial poloidal velocity, ω the local angular velocity and the integral is taken over the fluid volume. Viscous dissipation is neglected. We assume that the poloidal outflow in the radial direction takes place in the vicinity of the disc where $\omega \approx \omega_d$ below the disc, and that the inflow takes place near the bottom where ω has been reduced to a value $\beta\omega_d$ ($\beta < 1$). Then, the power becomes

$$W_t \approx \pi\varrho(1-\beta^2)\omega_d{}^2 r_2^4 \bar{v}_p/48. \tag{13}$$

Eq. (9), (11) and (13) give the result

$$W_t \approx \pi\varrho^2(1-\beta^2)^2\omega_d^4 r_2^5/2304\sigma\bar{B}^2. \tag{14}$$

About half-way between disc and bottom and at a distance $r_2/2$ from the axis the radial poloidal field has the relative magnitude

$$b_{pr}/B \approx b_{pr}/\bar{B} \approx \mu I_t/2r_2\bar{B} \approx \mu\varrho(1-\beta^2)(\omega_d r_2)^2/192\,\bar{B}^2, \tag{15}$$

and the axial poloidal component b_{pz} is likely to be small in the same region. The results (14) and (15) are not valid in the limits $\bar{B}=0$ and $\sigma=0$, where viscous dissipation must be taken into account.

V. Discussions

1. *The power input*

In the first experiment described in section III the angular velocity was $\omega_d = 42$ sec^{-1} and the field $B_d = 0.02$ Vs/m^2. Eq. (4), (6) and (7) give a power input $W_p = 5.8$

watts for the poloidal currents $\mathbf{i}_p$. With $\bar{B} = 0.03$ Vs/m^2 the toroidal currents require a power $W_t = 25$ watts if we put $\beta = 3/4$ in eq. (14). The result is not changed very much if the angular velocity is reduced by more than 25 % at the bottom ($\beta < 3/4$). The total power input becomes $W_m = W_p + W_t = 31$ watts which is of the same order as the measured value $W_m = 116$ watts. These estimations are valid, provided that the turbulence is not too intense, as may be expected when a smooth copper disc is used.

When changing to the radial strips, however, the power input increased about 300 times in absence of the magnetic field: the turbulent intensity was enhanced considerably. The external magnetic field was observed to decrease the power input, which may be explained by a suppression of turbulence as also has been observed in some earlier investigations (Lehnert 1952, 1955).

2. *The probe measurements*

According to the probe measurements the relative magnitudes of the induced toroidal and poloidal fields do not exceed the values $b_t/B = 0.25$ and $b_{pr}/B = 0.1$. With $\omega_d = 38.8$ sec^{-1}, $\bar{B} = 0.02$ Vs/m^2 and β put equal to 3/4 the estimations by eq. (5), (8) and (15) give the values $b_t/B \approx 0.29$ and $b_{pr}/B \approx 0.39$, which are of the same order as the experimental ones.

The agreement is as good as can be expected with the present method of measurement, where the motion is disturbed considerably in the vicinity of the probe. Further, no magnetic flux change would be indicated in a fluid of infinite conductivity; the measurements are only possible at finite conductivity where the field is able to diffuse across the probe windings.

The field patterns of Fig. 3 which are obtained from the probe measurements agree with the patterns to be expected from the elementary theory. A circulating poloidal motion $\mathbf{v}_p$ bends the field lines outwards in the upper and inwards in the lower parts of the vessel. It also provides a convection of angular momentum of the rotating fluid in the radial direction and may explain the observed increase in the twisting of the field lines in the lower layers as compared to the upper ones. The oppositely directed toroidal field $\mathbf{b}_t$ in some regions close to the wall does not imply a rotation in opposite direction. It may be explained by radial convection of angular momentum which causes the fluid to rotate faster at the point P in Fig. 3 than in the layers above P.

3. *The dynamo problem*

In Fig. 4 is shown a simple self-exciting dynamo which consists of a disc of radius r_1 which rotates with an angular velocity ω_d in a magnetic field with the strength B_d in the vicinity of the disc. The resistance around the circuit becomes

$$R \approx 2(\pi r_2 + d_2)/\sigma r_2^2 \tag{16}$$

and the magnetic field generated by the total current I is

$$B_d \approx B \approx \mu I / 2 r_2. \tag{17}$$

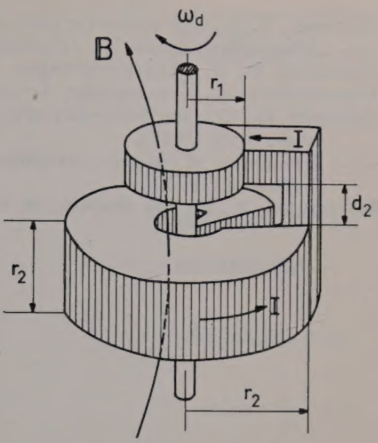

Fig. 4. A self-exciting dynamo (cf. Bullard and Gellman 1954). The magnetic field **B** is generated by a disc which rotates with the angular velocity ω_d.

Eq. (4) gives the induced electromotive force and from eq. (16) and (17) the angular velocity for self-excited operation becomes

$$\omega_d = 8(\pi r_2 + d_2)/\mu\sigma r_1^2 r_2. \tag{18}$$

With dimensions corresponding to Fig. 3 we obtain $\omega_d = 315\ \mathrm{sec}^{-1}$ for liquid sodium.

A fluid dynamo necessarily has to be asymmetric in order to become self-exciting. Thus, it is plausible that no such dynamo can exist in the configuration of Fig. 1 which has a low degree of asymmetry and has been operated at angular velocities not exceeding 47 sec^{-1}.

4. *Instability of a twisted magnetic field*

As suggested by Alfvén (1950) and proved by Lundquist (1951) a uniformly twisted magnetic field becomes unstable when the toroidal field is of the same magnitude as the axially directed, external field. However, the twisting in the present experiment has been somewhat too weak to cause such an instability, which, according to section IV, is expected to occur at least at five times higher speeds of rotation. This could be arranged by using a vessel completely filled with sodium, which prevents the formation of a cavity by which the free surface is bent down to the level of the disc. Further, the poloidal fluid motion should be suppressed, since it pushes the magnetic field out of the disc and reduces the effectiveness of the twisting.

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